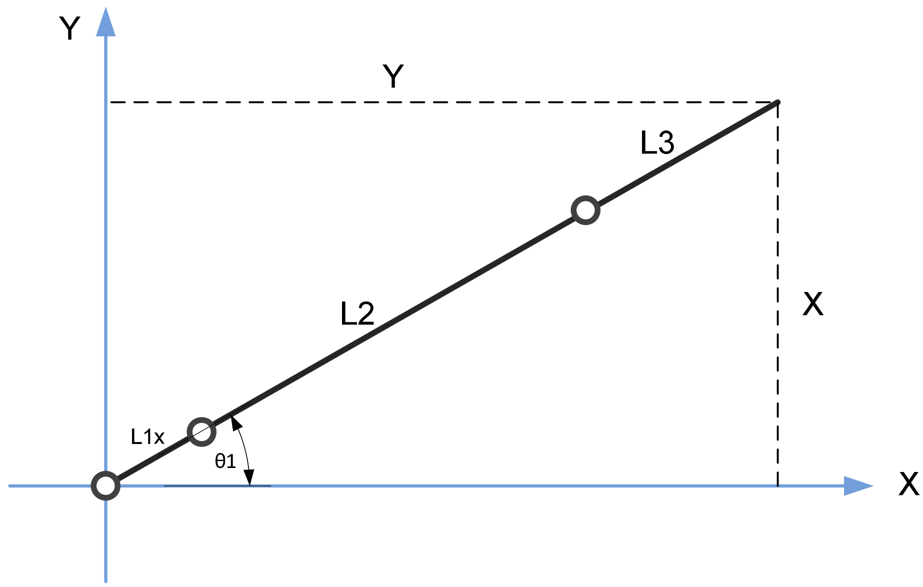

Figure 1: XY Plane (Top)

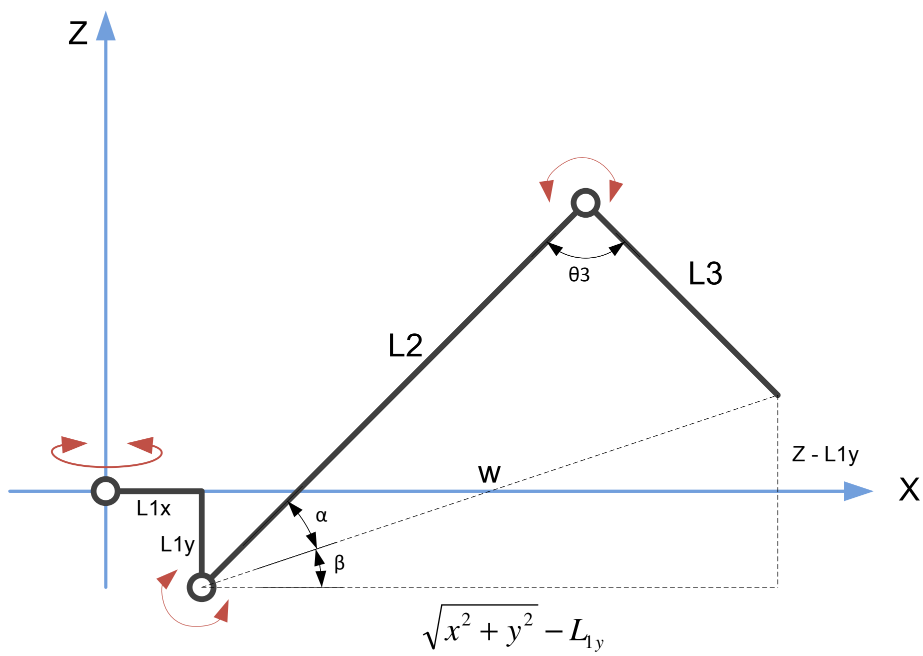

Figure 2: XZ Plane (Side)

SOLUTION:

θ_1 is the easiest to solve. Referring to figure 1, just use the Pythagorean Theorem.

$$\boxed{\theta_1 = atan(y, x)}$$

For θ_2 and θ_3, refer to figure 2. The solution uses the Law of Cosines for solving $cos\alpha$ and $cos\theta_3$. The Pythagorean Identities were also used.

$$w = \sqrt{\left(x - L_{1_x} cos\theta_1\right)^2 + \left(y - L_{1_x} sin\theta_1\right)^2 + \left(z - L_{1_y}\right)^2}$$

$$cos\alpha = \frac{{L_2}^2 + w^2 - {L_3}^2}{2L_2 w}$$

$$cos\theta_3 = \frac{w^2 - {L_2}^2 - {L_3}^2}{2L_2 L_3}$$

$$\beta = atan\left(z - L_{1_y}, \sqrt{x^2 + y^2} - L_{1_x}\right)$$

$$\alpha = atan\left(\sqrt{1 - cos\alpha^2}, cos\alpha\right)$$

$$\boxed{\begin{gathered} \theta_2 = -(\beta + \alpha) \\ \theta_3 = atan\left(\sqrt{1 - cos{\theta_3}^2}, cos\theta_3\right) \end{gathered}}$$

Note:

$$\alpha = atan(sin\alpha, cos\alpha)$$

using Pythagorean Identities:

$$sin\alpha = \sqrt{1 - cos\alpha^2}$$

therefore:

$$\alpha = atan\left(\sqrt{1 - cos\alpha^2}, cos\alpha\right)$$