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PREDICTION OF CONCENTRATION DISTRIBUTION IN PARABOLIC TROUGH SOLAR COLLECTORS

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The rational design of a parabolic trough solar collector PTC from metrological and material data necessitates the availability of a method for evaluating the optical performance. For this purpose, it is imperative to have precise knowledge of the concentrated flux pattern on the outer surface of the absorbing tube. The existing methods in literature are very general and the consequent expertise and computation facilities required to exercise them may prevent their use by the design engineers in industry.

In the present paper, a simple but reliable analytical method is developed which enables the determination of the concentrated flux distribution on the outer surface of the absorber tube of a PTC. Both defaults resulting from contours and sun tracking are considered in the analysis. A case study for a site having a latitude angle of 31° N is presented. The effects of collector and absorber tube dimensions as well as the contour and sun tracking errors on concentrated flux pattern, average flux concentration ratio and intercept factor are discussed.

KEY WORDS: Concentration distribution, Parabolic troughs

INTRODUCTION

Parabolic trough solar collectors (PTCs) have drawn remarkable attention for moderate temperature applications (100–400°C) such as the provision of industrial process heat and solar cooling [1–5]. This is accounted by their simplicity, reliability and lower unit cost. A cross section of a PTC is shown in Figure 1. The collector (a) is continuously tracking the sun so that the majority of incident sun rays on the collector surface are reflected to the absorber tube (b). The latest transfers a large portion of the absorbed radiation to the working fluid flowing through the tube.

Evans [6] has clearly and conclusively expressed the essential aspects of the optics of the PTC which relied upon the "cone optics". Nicolas [7] extended the understanding of the fundamental concentrated flux distribution by consideration of off-normal operation which is unavoidable with PTCs. The available computer models which may be adaptable to PTCs are very complicated and need a high expertise and special computation facilities, such as the model given in [8]. Jeter [9] developed a technique for evaluating the concentrated flux density distribution in a PTC by a semifinite formulation. The derivation of this technique appears to be difficult to follow and its use requires relatively long computation time. In [10] a simple method has been devised for predicting the concentrated flux distribution in a parabolic dish collector (paraboloid). This method can also be applied to a PTC as it is described briefly in the present work. In this method the non-ideal condition, to be expected in reality, has been taken into

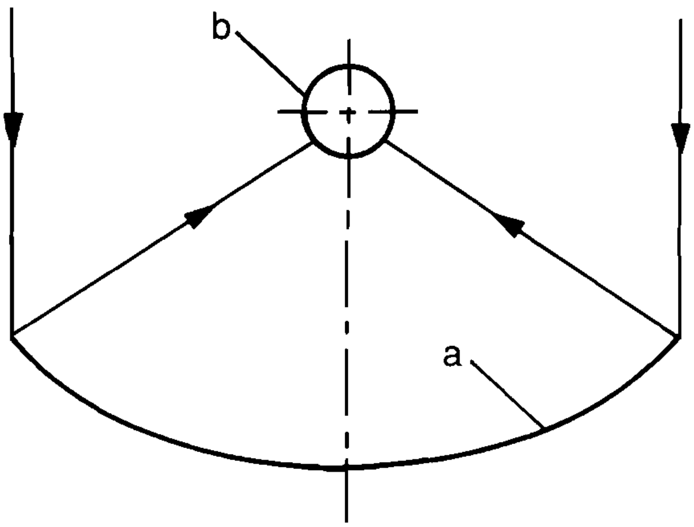

Figure 1 Cross section of a parabolic trough collector.
a collector contour *b* absorber tube

consideration. Of many possible imperfections in real collectors, two have been selected which affect considerably the concentrated flux on the absorber outer surface. These concern flaws in the contours of the PTC and sun tracking. Both kinds of flaws considered in this paper are of purely deterministic nature. Other flaws statistically distributed on the collector surface [11–14] do not affect considerably the flux pattern on the absorber outer surface but must be taken into account in thermal performance analysis. Thus, the presented method simulates approximately a real PTC.

The main characteristics of this method are: The simple derivation, and it needs relatively short computation time. Accordingly, it can be used by any engineer for practical calculation of the concentrated flux distribution in PTCs.

ANALYSIS

The following analysis is made for both ideal and real PTCs. All transverse contours of an ideal PTC are described by an equation of a parabola while those of a real PTC deviate more or less from parabola equation. Figure 2 shows a half cross-section of an ideal PTC (a) and its corresponding real one (b). A cartesian coordinate system (y,z) is set in the vertex of the shown cross section in which the z-axis coincides with the optical axis of the chosen contour. A point P laying on the ideal collector surface has the coordinates y and z which obey the equation of a parabola. In [15] an equation was derived that describes the points of the real collector surface, e.g point P' with coordinates y and z' as

$$z' = \frac{y^2}{4f} + 2\,\Delta z_{max}\,\frac{|y|}{w} \tag{1}$$

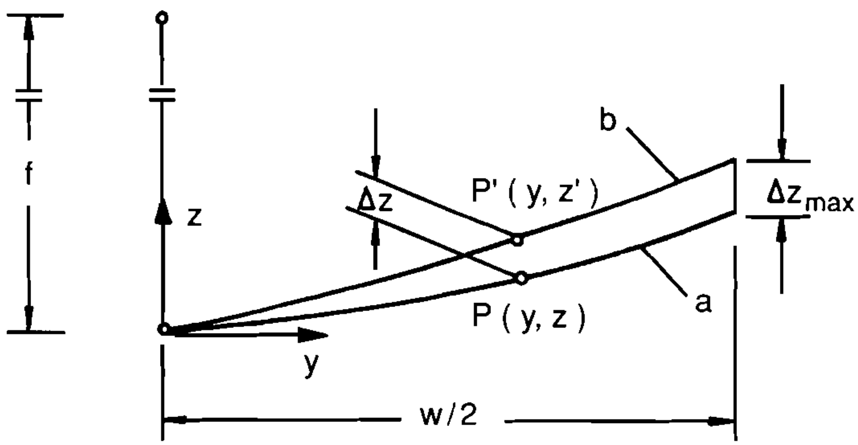

Figure 2 Contours of ideal and real collectors
a idea collector *b* real collector

In obtaining this equation, it was assumed that the deviations in the z-coordinates run linearly with the distance from the collector axis, i.e. the absolute value $|y|$ of the y-coordinate.

Consider a real parabolic collector of width w, length l and focal length f as sketched in Figure 3. A cartesian coordinate system (x, y, z) is set at the vertex of the lower transverse contour of the drawn collector in which the z-axis coincides with the optical axis of this contour and the x-axis lies in line with the collector axis. A plane ξ is located at random position in space above the collector. It may, for example, be tangential to the absorber surface. Its position in accordance with the coordinate system is described by its normal unit vector $\vec{n}_\xi$ and a point on it, as point C whose coordinates are x_c, y_c and z_c. An incident radiation cone with an aperture angle 32' is shown in Figure 3. Its central ray is inclined at an angle ε (sun tracking error) to the collector central plane, which is composed of the collector axis and focal-line, and makes an angle θ (incident angle) with the normal of the collector aperture i.e., the z-axis. The incident radiation cone is reflected by the collector surface at point P and generates in plane ξ an elliptical image of the sun. The coordinates of point P are x_p, y_p and z_p. The determination of both the unit vectors $\vec{s}_i$ and $\vec{s}_r$, in direction of the incident and reflected central rays, respectively, and the incident angle θ for specific collector parameters, at any solar time is reported in [15]. The point C on the plane ξ is irradiated by the reflected radiation cone only when it lies within the generated ellipse area. This is fulfilled when the angle ζ between the reflected central ray and the line $\overline{PC}$ is in the range

$$0' \leq \zeta \leq 16' \tag{2}$$

An infinitesimally small area dA_c, assumed to lie on the x-y plane, receives a radiation power given as:

$$dQ = q_s \, dA_c \cos\theta \tag{3}$$

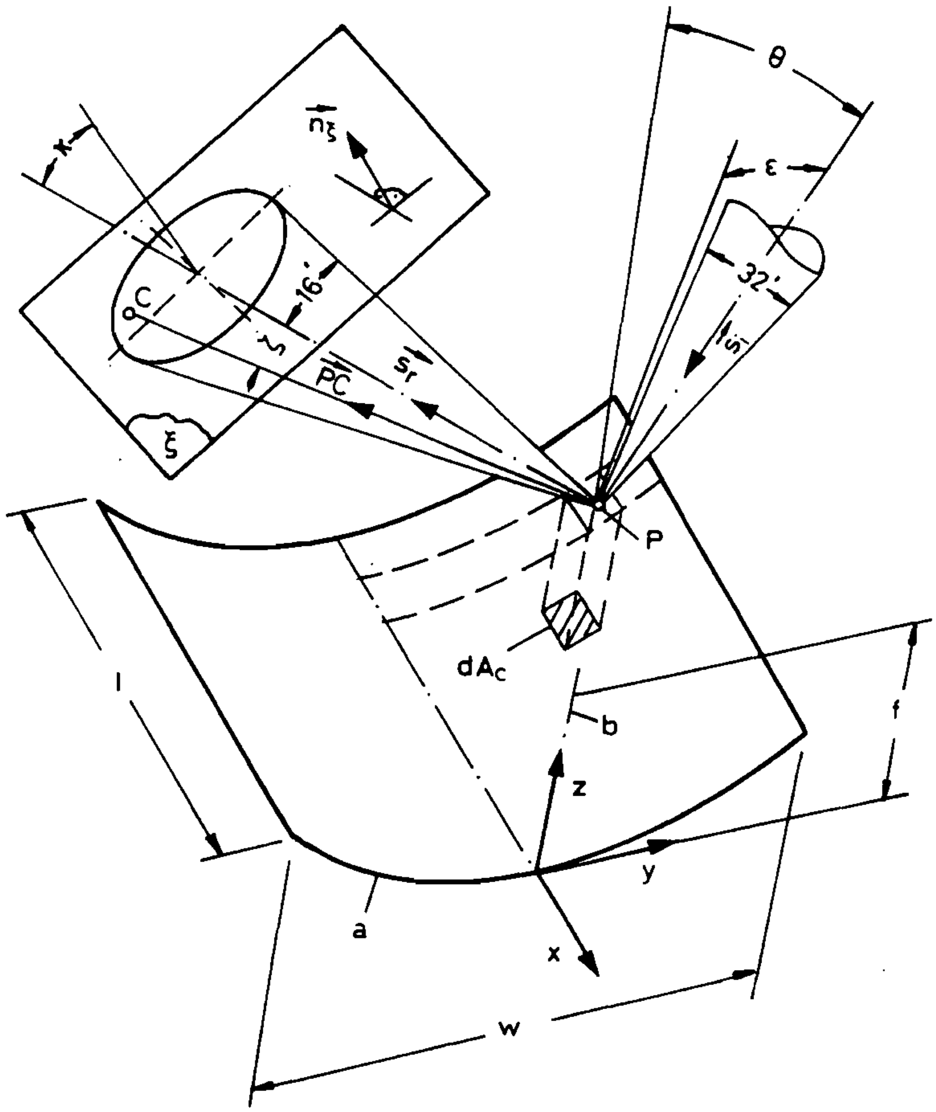

Figure 3 Determination of solar radiation intensity onto a point
a PTC *b* optical axis of the lower contour

where, q_s is the direct normal solar flux.

If the reflectivity of the collector surface for short-wave solar radiation is represented by ρ_r, the following solar energy is reflected from the collector:

$$dQ_r = \rho_r \, q_s \, dA_c \cos\theta \tag{4}$$

The radiant flux reflected by point P on the collector and incident on point C normal to ξ plane has been obtained in [10] as:

$$q_{C,P} = \frac{\rho_r \, q_s \cos\theta \cos\kappa}{\pi \, (\overline{PC} \cos\zeta \tan 16')^2} \, dA_c \tag{5}$$

Where, $\overline{PC}$ is the distance between points P and C, and κ is the angle between the normal to the plane ξ and the reflected central ray. The length $\overline{PC}$ is obtained with the aid of Figure 3, as:

$$\overline{PC} = \sqrt{(x_p - x_c)^2 + (y_p - y_c)^2 + (z_p - z_c)^2} \tag{6}$$

To determine the angle ζ, the unit vector $\overrightarrow{PC}$ in the direction from point P to point C must first be known. It is obtained in component notation as:

$$\overrightarrow{PC} = \frac{1}{\overline{PC}} \, (x_c - x_p, \, y_c - y_p, \, z_c - z_p) \tag{7}$$

The angle ζ can be calculated from:

$$\cos\zeta = \vec{s}_r \, \overrightarrow{PC} = \frac{1}{\overline{PC}} \, [s_{r,x} \, (x_c - x_p) + s_{r,y} \, (y_c - y_p) + S_{r,z} \, (z_c - z_p)] \tag{8}$$

Let the normal unit vector $\vec{n}_\xi$ of the plane ξ be defined as:

$$\vec{n}_\xi = (\, n_{\xi,x} \, , \, n_{\xi,y} \, , \, n_{\xi,z} \,) \tag{9}$$

By analogy with equation (8), the angle κ is then obtained as:

$$\cos\kappa = \vec{s}_r \, . \, \vec{n}_\xi = (\, s_{r,x} \, n_{\xi,x} + s_{r,y} \, n_{\xi,y} + s_{r,z} \, s_{\xi,z} \,) \tag{10}$$

Now the solar radiation intensity $q_{c,p}$ reflected from a point P on the collector surface onto point C on the plane ξ can be determined with aid of equation (5) if the normal unit vector $\vec{n}_\xi$ of a plane at a random location above the collector is known. With respect to the absorber, this implies that the unit vector of the irradiated absorber surface must be known. Figure 4 shows a plan view and a transverse section through the absorber collector arrangement. The vertex of the collector lower contour is again located at the origin of a cartesian coordinate system (x,y,z). The absorber is located with its axis coinciding with the collector focal line. It has an outside diameter d and a length l equal to the collector length. Furthermore, the central angle ϕ which progresses counterclockwise from the z axis is introduced, as shown in Figure 4. Given the x-coordinate and the angle ϕ for the point C, one can deduce the y and z coordinates as:

$$y_c = \frac{-d}{2} \sin\phi \, , \tag{11}$$

$$z_r = f + \frac{d}{2} \cos\phi \tag{12}$$

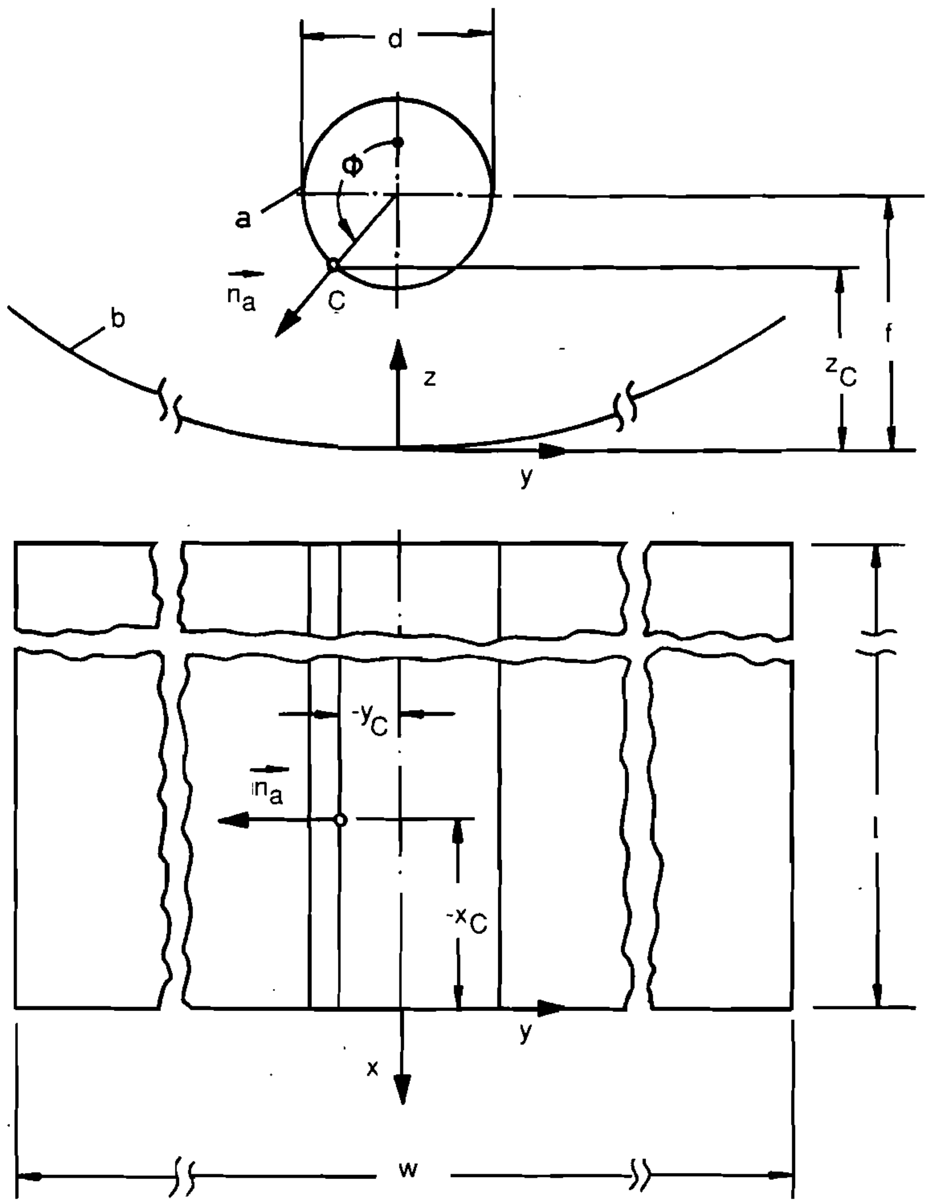

Figure 4 Unit vector normal to absorber surface
a absorber *b* collector

The equation of the normal unit vector $\vec{n}_a$ of the absorber is defined in component notation as:

$$\vec{n}_a = (0, -\sin\phi\ , \cos\phi\ ) \tag{13}$$

The variables of equation (5) are now known, and the solar radiation flux $q_{c,p}$ reflected from a point P on the collector surface onto a point C on the absorber surface can be calculated directly. Determination of the total solar radiation intensity q_c at point C necessitates integration over the collector surface. It is to be borne in mind that the absorber shades a part of the collector surface, the width of this shaded area is roughly equal to the absorber outside diameter d. Thus, the effective surface area of the collector of width w, becomes:

$$A_{c,e} = (w - d)\ l \tag{14}$$

Now the total solar radiation intensity q_c at point C is obtained with the aid of equation (5) as:

$$q_c = \frac{\rho_r\, q_s \cos\theta}{\pi \tan^2 16'} \int_{A_{c,e}} \frac{\cos\kappa}{(\overline{PC}\cos\xi)^2}\, dA_c \tag{15}$$

Where the reflectivity ρ_r of the entire collector surface is assumed to be uniform. Integration of equation (15) is performed numerically to calculate the insolation for any point on the absorber outer surface.

Since the concentrated flux intensity is proportional to the incident flux intensity on the reflector, it is preferable to compute the local flux concentration ratio

$$CF = \frac{q_c}{q_s} \tag{16}$$

over the receiver surface as this is a more flexible and useful measure of the collector optical performance. When needed, e.g. for heat transfer calculations, the local energy flux can be evaluated by multiplying the concentration ratio by the incident irradiance. For most purposes, it is desirable to compute the average flux concentration ratio $\overline{CF}$ as follows:

$$\overline{CF} = \frac{1}{2\pi} \int_{\phi=0}^{\phi=2\pi} CF\ d\phi \tag{17}$$

In this connection it is necessary to differentiate between the average flux and geometric concentration ratio. The latest is used to describe the relative size of the absorber. The geometric concentration ratio is defined for a parabolic trough collector with a tubular absorber as:

$$CR = \frac{w}{\pi\, d} \tag{18}$$

A procedure for calculating the optimum CR of both ideal and real PTC and for any incident angle is given in [15]. The optimum CR means the greatest CR which makes the absorper recives all reflected radiation. If CR is chosen greater than that, some reflected rays will pass by the absorber and do not impinige upon its surface. In this case the intercept factor γ is of interest in collector design. γ is defined as the ratio of the intercepted radiant energy by the absorber to the total reflected energy. Assuming that the collector length is so greater than its width, that the end effects of the absorber can be neglected, γ may be given by:

$$\gamma = \frac{d}{2\, \rho_r w \cos\theta} \int_{\phi=0}^{\phi=2\pi} CF \, d\phi \tag{19}$$

RESULTS AND DISCUSSIONS

The concentrated flux pattern on the outer surface of a tubular absorber with a parabolic trough collector depends in essence on the ratio f/w of the focal length to collector width, and geometric concentration ratio CR. Moreover it depends on the magnitude of both the collector contour and sun tracking errors. Concerning f/w, it has been revealed in [15] that the maximum geometric concentration ratio CR is obtained at f/w = 0.25. Most of the commercially constructed PTC, have values of f/w around 0.25 [1–5]. Therefore all the following results have been obtained for a PTC having f/w = 0.25. Besides, it has been assumed that the collector surface reflectivity for short-wave radiation is $\rho_r = 0.9$. The absorber axis is postulated as being located coincident with the collector focal line. The following results have been obtained for any site having a latitude of 31°N.

To study the effect of the sun tracking and collector contour errors, Figure 5 shows the local flux concentration ratio CF on the outer absorber surface for an incident angle $\theta = 0°$. In the left-hand diagram, perfect parabolic contour is considered for all transverse sections of the collector; i.e. $\Delta z_{max} / w = 0$. ε is chosen to have the values of 0', 15', 30' and 45'. For the right-hand diagram, the curve parameter $\Delta z_{max} / w$, defined as the ratio of the maximum contour error to the collector width, is chosen to have the values of –0.004, –0.002, 0.002 and 0.004 while ε is assigned zero for all curves of this diagram (fully precise sun tracking). For better understanding, the variation of CF pattern with the error parameters ε and $\Delta z_{max} / w$, Table 1 shows the geometric concentration ratio CR, obtained by using the procedure given in [15], for each curve of the two diagrams in Figure 5. This figure clearly shows that both the arc angle of the irradiated absorber surface and the concentrated flux decrease as ε increases. This is accounted by the decrease in the geometric concentration ratio CR as it can be seen from Table 1. It is to be mentioned here that the curves of CF have irregular change in some ranges of the angle ϕ (e.g. for $\varepsilon = 0'$ it occurs between $\phi \approx 150°$ and 210° and for $\varepsilon = 45'$ between $\phi \approx 160°$ and 300°). This is caused by the shadow resulting from the absorber itself on the collector surface.

Table 1. Effect of error parameters on geometric concentration ratio CR ($\theta = 0°$)

$\Delta Z_{max} / W$	ε *(minutes)*	*CR*
0	0	70.0
0	15	37.2
0	30	24.2
0	45	18.7
±0.002	0	50.3
±0.004	0	30.8

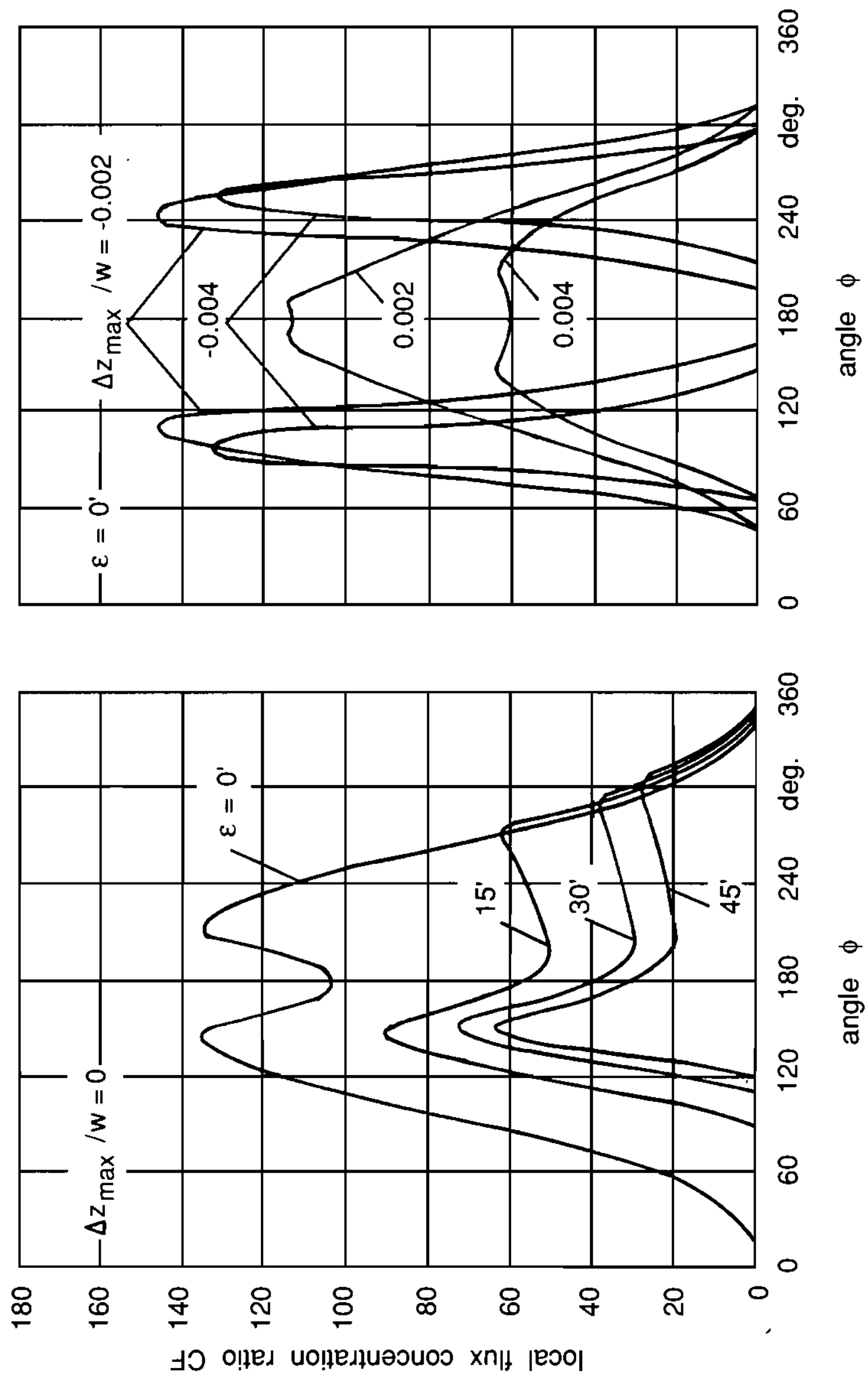

Figure 5 Concentrated solar radiation distribution on the outer surface of the absorber (f/w = 0.25, θ = 0°).

Investigating the right-hand diagram of Figure 5, one concludes that, an increase in $\Delta z_{max} / w$, in case of its positive values, leads to a decrease in both the arc angle of irradiated absorber surface and concentrated flux. The negative values of $\Delta z_{max} / w$ make the reflected radiation be concentrated on two separate regions of the absorber outer surface (e.g. for $\Delta z_{max} / w = -0.004$, the first region lies between $\phi = 70°$ and 170° and the second one extends from $\phi = 200°$ to 300°). The arc angles of irradiated collector surface as well as CF become smaller as $\Delta z_{max} / w$ decreases.

In Figure 6 the concentrated flux patterns are shown for the three chosen cases; $\varepsilon = 0'$. $\Delta z_{max} / w = 0$ (left-hand diagram); $\varepsilon = 30'$, $\Delta z_{max} / w = -0.004$ (middle diagram); and $\varepsilon = 30'$, $\Delta z_{max} / w = 0.004$ (right-hand diagram). The curve parameter is the incident angle θ which has the values of 0°, 22.5° and 45°. It is clear from this figure, that the arc angle of the irradiated absorber surface is slightly affected by θ, while the concentrated flux is significantly decreasing as θ increases. This is a direct consequence of the reduction in the incident solar irradiance on the collector surface due to the cosine factor loss and the decrease in the geometric concentration ratio as depicted in Table 2. It has been revealed in [15] that the geometric concentration ratio

Table 2. Effect of incident angle θ on geometric concentration ratio CR

$\Delta Z_{max} / W$	ε *(minutes)*	θ *(degrees)*	*CR*
0	0	0	70.0
		22.5	66.4
		45.0	50.8
±0.004	30	0	21.7
		22.5	21.4
		45.0	20.3

CR is dependent on the incident angle θ of the sun rays relative to the collector aperture. It increases with the decrease of θ and has its maximum at $\theta = 0°$. In order to ensure that all reflected sun rays by the PTC surface impinge upon the absorber outer surface (i.e. $\gamma = 1$), CR should be chosen for the greatest possible θ during the collector operation period. In this case the absorber diameter has its greatest value. Considering that PTC is used 7 hours per day around noon time, the value of 45° for the maximum possible θ is reasonable for the different collector orientations.

Figure 7 shows the concentrated flux patterns for three absorber diameter ratios; i.e. $d / d_{45} = 0.6$, 0.8 and 1.0. Hereby d_{45} is the least absorber diameter for which $\gamma = 1$, $\theta = 45°$, $\varepsilon = 30'$ and $\Delta z_{max} / w = \pm 0.004$. The figure represents three cases of the PTC: $\varepsilon = 0'$, $\Delta z_{max} / w = 0$ (left-hand diagram); $\varepsilon = 30'$, $\Delta z_{max} / w = -0.004$ (middle diagram); and $\varepsilon = 30'$ $\Delta z_{max} / w = 0.004$ (right-hand diagram). This figure shows that the arc angle generally decreases while the flux intensity increases with the decrease of d specially for ideal collector.

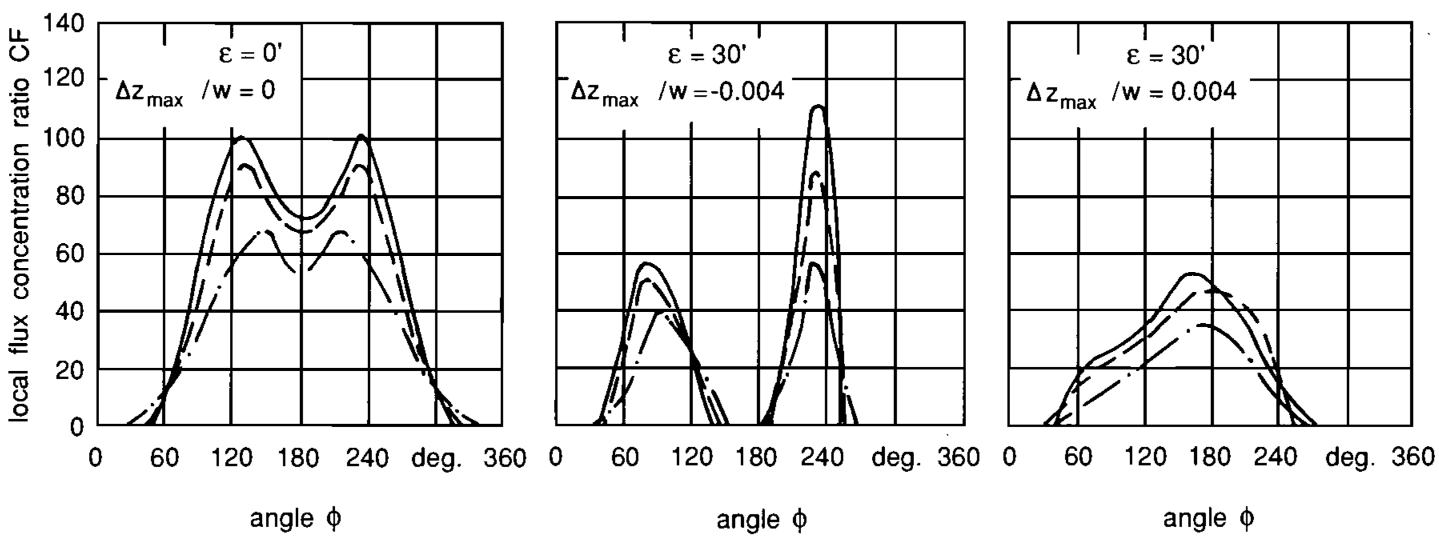

Figure 6 Effect of incident angle on concentration distribution on the absorber surface ($f/w = 0.25$)
——— $\theta = 0°$ — — — $\theta = 22.5°$ —.— $\theta = 45°$

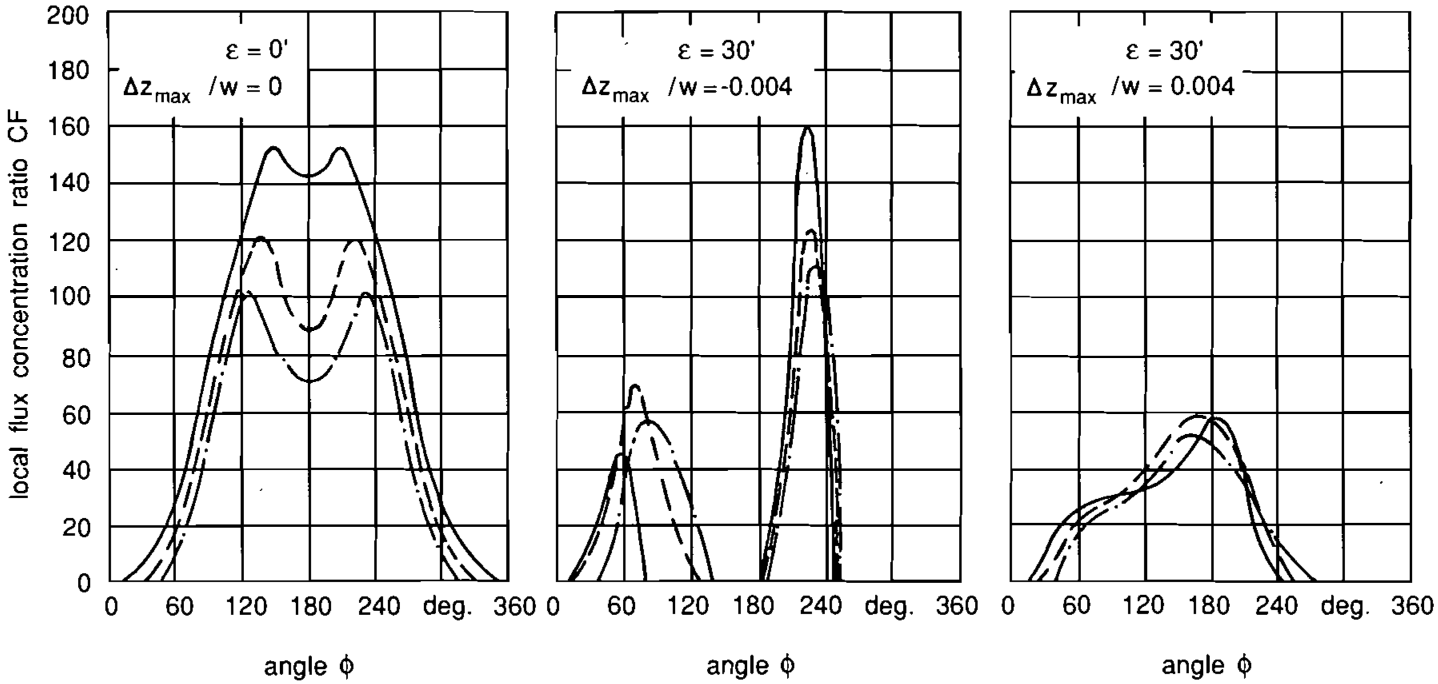

Figure 7 Effect of absorber diameter on concentration distribution on the absorber surface ($f/w = 0.25$, $\theta = 0°$)
——— $d/d_{45} = 0.6$ — — — $d/d_{45} = 0.8$ — . — $d/d_{45} = 1.0$

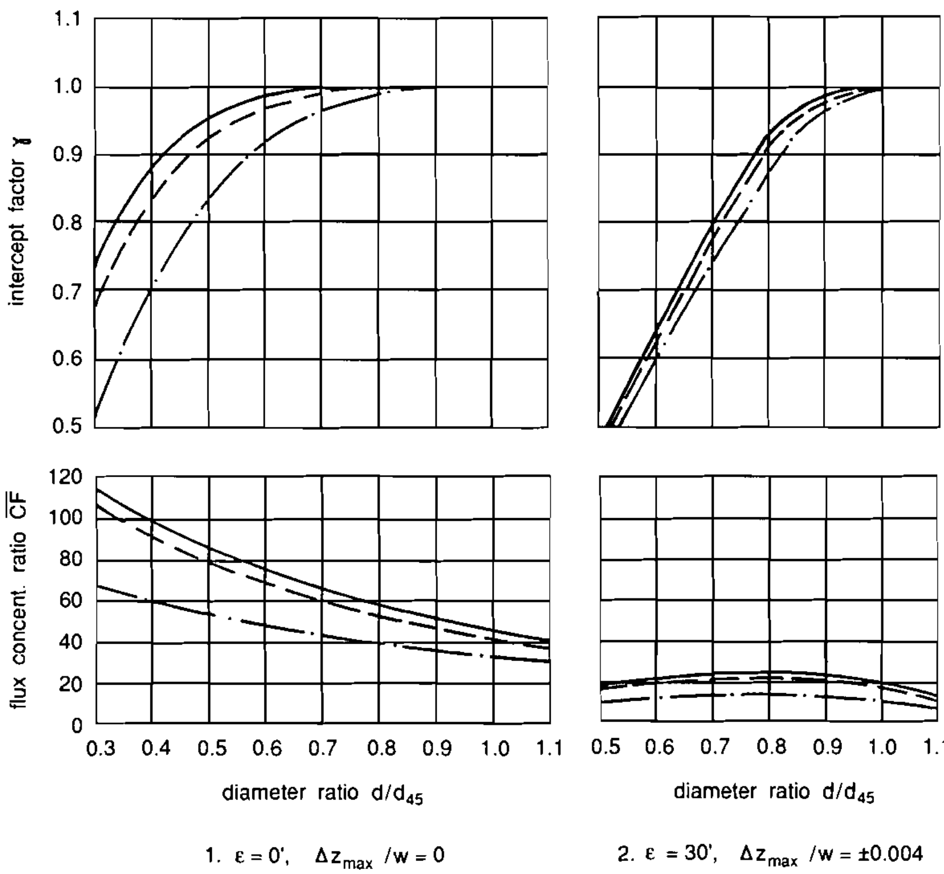

Figure 8 Dependence of intercept factor and average flux concentration ratio on absorber diameter.
—— $\theta = 0$ — — — $\theta = 22.5°$ —.— $\theta = 45°$

The average flux concentration ratio $\overline{CF}$ and the intercept factors γ have been calculated by aid of Figure 7 and use of equations (18) and (19) respectively . γ and $\overline{CF}$ are plotted in the upper and lower diagrams of Figure 8 respectively, for $\theta = 0°$, 22.5° and 45°. It is to be observed here that the sign of Δz_{max} / w has a very small unremarkable effect on both γ and $\overline{CF}$. It is obvious from Figure 8 that the intercept factor increases sharply with the increase of the diameter ratio, reaching a unity as the absorber diameter d is getting closer to the diameter d_{45}. As is shown in both diagrams, the range of the diameter ratio at which $\gamma = 1$ is greater in case of ideal collector system, (ε = 0', Δz_{max} / w = 0) than that of real one (ε = 30' Δz_{max} / w = ±0.004). γ decreases by 10% at d/d_{45} = 0.42 and 0.78 for each case respectively when $\theta = 0$. $\overline{CF}$ increases very sharply as d decreases in case of ideal collector system, while it is slightly decreasing with the increase of d/d_{45} in case of real one. Both γ and $\overline{CF}$ decrease with the increase of the incident angle θ mainly due to the cosine loss factor.

CONCLUSIONS

A solar concentrating system consisting of a parabolic trough collector and a tubular absorber, located with its axis in-line with the collector focal line, was investigated. For such system, a simple but reliable analytical method was presented, which enables the determination of the concentrated flux distribution on the absorber outer surface. By aid of this method calculations were made on an example of a site having a latitude of 31° N. The results reveal that, the increase of the following parameters; the collector contour error, sun tracking inaccuracy, incident angle and absorber diameter tends to reduce both the arc angle of the irradiated absorber surface and the concentrated radiation intensities as well. The intercept factor γ decreases slightly with the decrease in absorber diameter within some range, then it goes down sharply. This range is greater in case of ideal collector system than in real one. The average flux concentration ratio $\overline{CF}$ increases considerably in case of ideal system as the absorber diameter decreases, whereas $\overline{CF}$ is almost constant for the real collector system.

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NOMENCLATURE

$A_{c,e}$	:	effective surface area of the collector	m^2
CF	:	local flux concentration ratio	
$\overline{CF}$	:	average flux concentration ratio	
CR	:	geometric concentration ratio	
d	:	outer diameter of the absorber	m
d_{45}	:	the smallest diameter of the absorber for $\gamma = 1$ $\theta = 45°$, $\varepsilon = 30$ and $\Delta Z_{max} / w = \pm 0.004$	m
f	:	focal length	m
l	:	collector length	m
$\vec{n}'_a$	:	normal unit vector of the absorber surface	m
n_ξ	:	normal unit vector of the plane ξ	m
q_c	:	concentrated radiation intensity at point C	kW/m^2
q_s	:	normal solar radiation irradiance	kW/m^2
x, y, z	:	cartesian coordinates with x-axis in the axial direction & z-axis is the optical axis	m
w	:	collector width	m

Greek letters

γ	:	intercept factor	
ΔZ_{max}	:	maximum contour error	m

ε	:	sun tracking error	rad.
θ	:	incident angle	rad.
ρ_r	:	reflectivity of the collector surface for solar radiation.	
ϕ	:	centreal angle of the absorber surface	

References

1. D. Jaffe, S. Friedlander and D. Kearney, "The LUZ Solar Electric Generating Systems in California", *Advances in Solar Energy Technology*, Pergamon Press (1988).
2. P. Wattiez. "DCS System Performance: October 1982 –September, 1983", DCS first term workshop, Almeria, Spain, December (1983).
3. C. Jensen, H. Preice, and D. Kearney, "The Segs Power Plants: 1988 Performance", ASME International Solar Energy Conference, San Diego, California, April (1989).
4. W. S. Duff and G. F. Lamerio, "A Performance Comparison Method for Solar Concentrators", ASME Publication 74-WA/Sol-4, Presented at ASME Winter Annual Meeting, New York, November (1974).
5. A. Rable, "Comparison of Solar Concentrators", *Solar Energy*, **21** (1978).
6. D. L. Evans, "On the Performance of Cylindrical Parabolic Solar Energy", **19(4)** (1977).
7. R. O. Nicolas and J. C. Duran, "Generalization of the Two-dimenstional Optical Analysis of Cylindrical Concentrators", *Solar Energy*, **25**, pp. 21–31 (1980).
8. F. Biggs and C. N., Vittitoe, "The HELIOS Model for the Optical Behaviour of Reflecting Solar Concentrators", Sandia National Laboratory, Report SAND 76–0347, March (1979).
9. S. M. Jeter, "Analytical Determination of the Optical Performance of Practical Parabolic Trough Collectors From Design Data", *Solar Energy*, **39(1)** (1987).
10. K. Bammert, A. Hegazy and H. Lange, "Determination of the Distribution of Incident Solar Radiation in Cavity Receivers with Approximately Real Parabolic Dish Collectors", Transactions of ASME, *J. of Solar Energy Engineering*, **112**, pp 237–243 (1990).
11. R. Köhne and J. Kleih, J. "About Power of High Concentrating Mirror Concentrators and Mirror Systems" (in German), Deutsche Forschungs und Versuchsanstalt Fur Luft-und Raumfahrth, FB 87–29, Koln, Germany (1987).
12. H. M. Güven, R. B. Bannerot and F. Mistree, "Determination of Error Tolerances for Optical Design of Parabolic Troughs", ASME paper 83 WA/Sol-1 (1983).
13. M. J. O'Neil, and Hudson, "Optical Analysis of Paraboloidal Solar Concentrators" Proceedings of the Annual Meeting of the American Sec. of the Int. Solar Energy Society, Denver, Colo., **21**, pp. 885–862 (1978).
14. A. C. Ratzel, B. D. Boughton, T. R. Mancini and R. B. Diver, "CIRCE: Computer Code for the Analysis of Point-Focus Solar Concentrators", Solar Engineering – 1987, Proceedings of the ASME-JSME-JSES Solar Energy Conference, Honolulu Hawaii, **1**, pp. 136–144 (1987).
15. A. S. Hegazy, M. A. Hassab and M. M. El-Kassaby, "A three Dimensional Model for Determination of the Concentration Ratio of a Parabolic Trough Solar Collector" Accepted for Publication in the Eighth International Conference of Mechanical Power Engineering, Alexandria, Egypt, April 27–29 (1993).